Local Linearity

The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

- to write the equation of the tangent line, we need a point and a slope!

point $(2,1)$ $\quad m=4$

$$y-1=4(x-2)$$
$$y=4x-7 \checkmark$$

$$f(1.9)=4(1.9)-7$$
$$=7.6-7$$
$$f(1.9)=0.6 \checkmark$$

$f(x) = \frac{1}{x-1}$ Find a linear approximation of f around $x = -1$.

$f(x) = \frac{1}{x-1} = (x-1)^{-1}$

$(-1, -\frac{1}{2})$ $m = f'(-1) =$

$f'(x) = -1(x-1)^{-2}$

$f'(-1) = \frac{-1}{4}$

$f'(x) = \frac{-1}{(x-1)^2}$

$y + \frac{1}{2} = -\frac{1}{4}(x+1)$

$4y + 2 = -(x+1)$

$4y + 2 = -x - 1$

$x + 4y = -3$

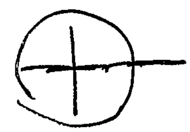
L' Hopitâl's Rule

- Use it with limits of indeterminate form!

$$\frac{0}{0}, \quad \frac{\pm\infty}{\pm\infty}$$

→ If $\lim\limits_{x\to c} f(x) = 0$ **AND** $\lim\limits_{x\to c} g(x) = 0$ **AND** $\lim\limits_{x\to c} \dfrac{f'(x)}{g'(x)} = L$.

then $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = L$.



Ex. 1

$$\lim_{x\to 0} \frac{\overbrace{\sin x}^{f(x)}}{\underbrace{x}_{g(x)}} \qquad \lim_{x\to 0} \frac{\cos x}{1} = 1$$

$$\Downarrow$$

$$\lim_{x\to 0} \frac{\cos x}{1} = 1$$

Ex. 2

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{2x}{1} = \frac{2}{1} = 2$$

Ex. 3

$$\lim_{x\to 0} \frac{2\sin x - \sin(2x)}{x - \sin x} = \frac{2(0) - 0}{0 - 0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{2\cos x - 2\cos(2x)}{1 - \cos(x)} = \frac{2(1) - 2(1)}{1 - 1} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{-2\sin x + 4\sin(2x)}{\sin x} = \frac{-2(0) + 4(0)}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{-2\cos x + 8\cos(2x)}{\cos x} = \frac{-2(1) + 8(1)}{1} = 6$$

$$\boxed{\lim_{x\to 0} \frac{2\sin x - \sin(2x)}{x - \sin(x)} = 6}$$

Ex. 4

$$\lim_{x\to \infty} \frac{4x^2 - 5}{1 - 3x^2} = \frac{\infty}{-\infty}$$

$$\lim_{x\to \infty} \frac{8x}{-6x} = \frac{\infty}{-\infty}$$

$$\lim_{x\to \infty} \frac{8}{-6} = -\frac{4}{3}$$

$$\boxed{\lim_{x\to \infty} \frac{4x^2 - 5}{1 - 3x^2} = -\frac{4}{3}}$$