



$$x^2 + y^2 = 1$$

Could  
split

$$y = \sqrt{1-x^2} \quad y = -\sqrt{1-x^2}$$

Instead use Implicit differentiation ...

$$\frac{d}{dx} [x^2 + y^2] = \frac{d}{dx} [1]$$

$$2x + 2y \frac{dy}{dx}$$

$$\frac{d}{dx} [y^2] = \frac{d}{dx} [(y(x))^2]$$

$$\frac{d}{dx} [x^2] + \frac{d}{dx} [y^2] = \frac{d}{dx} [1] \quad \text{really chain rule}$$

$$2x + \frac{d(y^2)}{dx} \cdot \frac{dy}{dx} = 0$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Ex. 1  $(x-y)^2 = x+y-1$

$$2(x-y) \cdot (1-y') = 1+y'$$

$$(2x-2y) \cdot (1-y') = 1+y'$$

$$\begin{array}{r} 2x - 2xy' - 2y + 2yy' = 1 + y' \\ \hline -2x \qquad \qquad \qquad +2y - y' \qquad \qquad \qquad -y' + 2y - 2x \end{array}$$

$$-2xy' - y' + 2yy' = 1 + 2y - 2x$$

$$\frac{y'(-2x - 1 + 2y)}{(-2x - 1 + 2y)} = \frac{1 + 2y - 2x}{(-2x - 1 + 2y)}$$

$$y' = \frac{1 + 2y - 2x}{-1 + 2y - 2x}$$

Ex. 2  $x^2 + (y-x)^3 = 28$

Evaluate  $\frac{dy}{dx}$  when  $x=1$ .

$$\begin{array}{r} 2x + 3(y-x)^2 \cdot (y'-1) = 0 \\ \hline -2x \qquad \qquad \qquad -2x \end{array}$$

$$\frac{3(y-x)^2 \cdot (y'-1)}{3(y-x)^2} = \frac{-2x}{3(y-x)^2}$$

$$y' - 1 = \frac{-2x}{3(y-x)^2} + 1$$

$$y' = \frac{-2x}{3(y-x)^2} + 1$$

$$y' = \frac{-2(1)}{3(4-1)^2} + 1 = \frac{-2}{27} + 1$$

When  $x=1$ ,  $y=4$

$$y'_{\text{when } x=1} = \frac{25}{27}$$

$$x^2 + (y-x)^3 = 28$$

$$1^2 + (y-1)^3 = 28$$

$$(y-1)^3 = 27$$

$$y-1 = 3$$

$$+1 \quad +1$$

$$y = 4$$